Using Cramer's Rule to Solve Systems of Linear Equations in Two Variables

Given two equations: $a_1x + b_1y = k_1$
$a_2x + b_2y = k_2$

a_1 a_2 are the coefficients of x
b_1 b_2 are the coefficients of y
k_1 k_2 are the constants

Step 1: Find the D, D_x, and D_y determinants by arranging the coefficients and constants in the following manner. You will need to enter these values as matrixes in Step 2, so write them down.

$$D = \begin{matrix} a_1 & b_1 \\ a_2 & b_2 \end{matrix} \qquad D_x = \begin{matrix} k_1 & b_1 \\ k_2 & b_2 \end{matrix} \qquad D_y = \begin{matrix} a_1 & k_1 \\ a_2 & k_2 \end{matrix}$$

Step 2: Enter D, D_x, and D_y as 2 x 2 matrixes ([A], [B], and [C]) using the matrix editor.

[2nd] [x-1] ► EDIT [1:] [A] to enter values in matrix [A]

Enter values by rows. For example, to store determinant D as matrix [A], enter the values in this order (press [ENTER] after each one): a_1 b_1 a_2 b_2

When you have finished entering values for one matrix, press [2nd] [Mode] to exit the editor. Invoke the matrix editor twice more, to enter the values for D_x, and D_y.

Step 3: Use the det ( ) feature to calculate each determinant (which is simply a number). You will use these values in Step 4, so write them down. Invoke the det ( ) feature as follows:

[2nd] [x-1] ► MATH [1:] det ([ENTER]
det (

Invoke the det ( ) feature three times, once for each matrix:

det ([2nd] [x-1] NAMES [1] [)] [ENTER] to find D (using the values stored in 1: [A])

det ([2nd] [x-1] NAMES [2] [)] [ENTER] to find D_x (using the values stored in 2: [B])

det ([2nd] [x-1] NAMES [3] [)] [ENTER] to find D_y (using the values stored in 3: [C])

Step 4: Apply Cramer's Rule to find the solution (that is, $\boldsymbol{x}$ and $\boldsymbol{y}$):

$$\boldsymbol{x} = \frac{D_x}{D} \quad \text{and} \quad \boldsymbol{y} = \frac{D_y}{D}$$

In other words, to find the value for the $\boldsymbol{x}$-coordinate of the solution, divide the value you got when you calculated det ([B]) by the value you got when you calculated det ([A]).

Similarly, to find the value for the $\boldsymbol{y}$-coordinate of the solution, divide the value you got when you calculated det ([C]) by the value you got when you calculated det ([A]).

NOTE: If you're feeling ambitious, assuming you entered values for D, D_x, and D_y as matrixes [A], [B], and [C], you can find $\boldsymbol{x}$ all in one step as follows:

[2nd] [x⁻¹] ► MATH [1] [2nd] [x⁻¹] NAMES [2] [)] [÷] [2nd] [x⁻¹] ► MATH [1] [2nd] [x⁻¹] NAMES [1] [)]

[ENTER]

Then find $\boldsymbol{y}$ as follows:

[2nd] [x⁻¹] ► MATH [1] [2nd] [x⁻¹] NAMES [3] [)] [÷] [2nd] [x⁻¹] ► MATH [1] [2nd] [x⁻¹] NAMES [1] [)]

[ENTER]

Step 5: Write the solution as an ordered pair ($\boldsymbol{x}$, $\boldsymbol{y}$).

Remember that the *solution* to a system of linear equations is a *point in the rectangular coordinate system*, specifically, the point where the two lines intersect.